AP Calculus BC

Unit 6 – Integration Techniques

AP Calculus BC – Worksheet 39 **Basic Integration/Initial Value Problems**

Evaluate each indefinite integral

1) $\int\left(2x^3-3x^{-4}+\sec^2 x\right)dx$	2) $\int(\cos 8x)\,dx$	3) $\int\left(9x^{-1/2}+9x^{1/2}\right)dx$
4) $\int\left(4x^{1/3}+2x^{-2/3}+6\right)dx$	5) $\int\left(\sec^2 x-9\right)dx$	6) $\int\left(e^{8x}+5x^{1/2}\right)dx$
7) $\int\frac{4x^4-15x^3}{x}dx$	8) $\int\frac{5+u}{u}du$	9) $\int\frac{36t^9-7}{t}dt$

10) Find the general solution to the exact differential equation: $\frac{dy}{dx}=-2\sin x-e^{-x}+7x^6$.

11) Find the particular solution to the exact differential equation: $\frac{ds}{dt}=-5+6\cos t;\ s(0)=2$.

12) Solve the initial value problem: $\frac{dy}{dx}=-4e^x+\sin x;\ y=-10$ when $x=0$.

13) Solve the initial value problem: $\frac{du}{dx}=6x^5-4x^3+4;\ u=-2$ when $x=1$.

14) Solve the initial value problem: $\frac{dx}{dt}=\frac{7}{t}-\frac{3}{t^4}-2;\ x=10$ when $t=1$.

15) Find the function F that satisfies the following differential equations and initial conditions:
$F''(x)=\cos x;\ F'(\pi)=5;\ F(\pi)=6$

AP Calculus BC – Worksheet 40 **"Most Complicated" Rule**

Find the indefinite integral

1) $\int (3x-4)^5\,dx$	2) $\int 6x^2\left(x^3+4\right)^5\,dx$	3) $\int \frac{\left(\sqrt{x}-1\right)^2}{\sqrt{x}}\,dx$
4) $\int \sin x\, e^{\cos x}\,dx$	5) $\int \cot(3x)\,dx$	6) $\int \frac{x^2}{\left(1+x^3\right)^2}\,dx$
7) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$	8) $\int \frac{\sin x}{1+\cos^2 x}\,dx$	9) $\int \frac{\sec^2 x}{\sqrt{\tan x}}\,dx$
10) $\int \frac{x\,dx}{\sqrt{1-x^2}}$	11) $\int \frac{(\ln x)^5}{x}\,dx$	12) $\int \frac{e^x}{4-e^x}\,dx$
13) $\int \frac{1}{25x^2+1}\,dx$	14) $\int \sec(2x)\tan(2x)\,dx$	15) $\int \cot x \csc^2 x\,dx$

Answers

1) $\frac{1}{18}(3x-4)^6+C$	2) $\frac{1}{3}\left(x^3+4\right)^6+C$	3) $\frac{2}{3}\left(\sqrt{x}-1\right)^3+C$
4) $-e^{\cos x}+C$	5) $\frac{1}{3}\ln\lvert\sin(3x)\rvert+C$	6) $\frac{1}{9}\left(1+x^3\right)^3+C$
7) $2\sin\sqrt{x}+C$	8) $-\arctan(\cos x)+C$	9) $2\sqrt{\tan x}+C$
10) $-\sqrt{1-x^2}+C$	11) $\frac{1}{6}(\ln x)^6+C$	12) $-\ln\lvert 4-e^x\rvert+C$
13) $\frac{1}{5}\arctan(5x)+C$	14) $\frac{1}{2}\sec(2x)+C$	15) $-\frac{1}{2}\cot^2 x+C$ $-\frac{1}{2}\csc^2 x+C$

AP Calculus BC – Worksheet 41 **Integration by *u*-Substitution**

Evaluate the indefinite integral by using the given substitution.

1) $\int \cos(6x)\,dx; \quad u = 6x$	2) $\int 63(9x-7)^{-8}\,dx; \quad u = 9x-7$	3) $\int 28r^6\left(7-r^7\right)dr; \quad u = 7-r^7$

Use substitution to find the indefinite integral.

4) $\int 12\left(y^4+4y^2+8\right)^2\left(y^3+2y\right)dy$	5) $\int \frac{5}{(5x-3)^2}\,dx$	6) $\int \sin(8z-9)\,dz$
7) $\int \frac{\ln^{14} x}{x}\,dx$	8) $\int \tan x\,dx$	9) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$
10) The function f is continuous and $\int_4^{19} f(u)\,du = 10$. What is the value of $\int_1^4 \left[x \cdot f\left(x^2+3\right)\right]dx$?		

AP Calculus BC – Worksheet 42 **The Definite Integral**

1) The functions f and g are integrable and $\int_2^4 f(x)\,dx = 6$, $\int_2^7 f(x)\,dx = 8$, and $\int_2^7 g(x)\,dx = 8$. Find the values of the following definite integrals:

a) $\int_4^4 f(x)\,dx$	b) $\int_7^2 g(x)\,dx$	c) $\int_2^7 9g(x)\,dx$
d) $\int_4^7 f(x)\,dx$	e) $\int_2^7 \left[g(x)-f(x)\right]dx$	f) $\int_2^7 \left[5g(x)-f(x)\right]dx$

2) Evaluate $\int_2^6 \frac{4}{x}\,dx$.	3) Evaluate $\int_0^5 \sqrt{y+4}\,dy$
4) Evaluate $\int_0^1 \left(t^5+4t\right)^{1/2}\left(5t^4+4\right)dt$	5) Evaluate $\int_3^7 \frac{18x\,dx}{9x^2+5}$
6) $\int_0^{\pi/4} \sin x\,dx =$	7) Evaluate $\int_0^{\pi/4} \frac{4\sin(4t)}{8-\cos(4t)}\,dt$
8) Use *u*-substitution to evaluate: $\int_0^{\pi/8} 5^{\cos 4t} \sin 4t\,dt$	9) Use *u*-substitution to evaluate: $\int_0^4 12x^2 e^{x^3}\,dx$
10) Evaluate $\int_0^{4/7} \frac{dx}{49x^2+16}$	

AP Calculus BC – Worksheet 43 **Integration by Parts #1**

1	$\int x\sin(6x)\,dx =$
2	$\int 9te^{2t} =$
3	$\int x^2e^{10x}\,dx =$
4	$\int x^2\cos(4x)\,dx =$
5	$\int 20x^2\ln x\,dx =$
6	Solve the initial value problem: $\frac{dy}{dx} = 9x\csc^2(3x)$; $y = 1$ when $x = \frac{\pi}{2}$.
7	$\int 6x^3e^{-5x}\,dx =$
8	$\int_0^{\pi/2}(x-2)\sin x\,dx =$
9	$\int_1^{e^2} x^5\ln(x)\,dx =$

AP Calculus BC – Worksheet 44 **Integration by Parts**

1. If $\int x^2 \cos x dx = h(x) - \int 2x \sin x dx$, find $h(x)$.

2. Evaluate $\int x \sin(5x) dx$.

3. Evaluate $\int x \csc^2 x dx$

4. Find the function y if $\frac{dy}{dx} = x \sec^2 x$ and $y = 1$ when $x = 0$.

5. Evaluate $\int_0^{\pi} t \sin 3t dt$

6. Evaluate $\int_0^1 (x^2 + 1) e^{-x} dx$

7. Evaluate $\int_1^e \frac{\ln x}{x^2} dx$

8. The table gives the values of f, f', g, and g' for selected values of x. If $\int_0^3 f'(x) g(x) dx = 6$, then $\int_0^3 f(x) g'(x) dx = ?$

x	0	3
$f(x)$	1	5
$f'(x)$	5	–3
$g(x)$	–4	3
$g'(x)$	3	2

9. Let f be a twice-differentiable function with selected values of f and its derivatives shown in the table. What is the value of $\int_0^3 x f''(x) dx$?

x	$f(x)$	$f'(x)$	$f''(x)$
0	2	–2	5
3	5	7	–2

AP Calculus BC – Worksheet 45 **Partial Fraction Decomposition**

1	Find the values of *A* and *B* that complete the partial fraction decomposition: $\frac{15x+93}{x^2+13+40}=\frac{A}{x+5}+\frac{B}{x+8}$
2	Find the values of *A* and *B* that complete the partial fraction decomposition: $\frac{4}{x^2-8x-20}=\frac{A}{x-10}+\frac{B}{x+2}$
3	Find the values of *A* and *B* that complete the partial fraction decomposition: $\frac{15x+75}{x^2+11x+24}=\frac{A}{x+8}+\frac{B}{x+3}$
4	Find the values of *A* and *B* that complete the partial fraction decomposition: $\frac{9x-17}{x^2-4x+3}=\frac{A}{x-3}+\frac{B}{x-1}$
5	Evaluate $\int\frac{1}{64-121x^2}dx$
6	$\int\frac{9}{x^2+3x}dx=$
7	$\int\frac{11}{2x^2-9x-5}dx=$
8	$\int_{1/2}^{1}\frac{2y+3}{y^2+y}dy=$
9	$\int_{4}^{5}\frac{4}{x^2+2x-3}dx=$

AP Calculus BC – Worksheet 46 **Improper Integrals**

Evaluate the integral or state that it diverges.

1) $\int_2^{\infty} \frac{5}{x^2}\,dx$	2) $\int_{-\infty}^{-27} x^{-1/3}\,dx$
3) $\int_{-\infty}^{-1} -2x^{-2}\,dx$	4) $\int_2^{\infty} \frac{4}{x^2+5x+6}\,dx$
5) $\int_{15}^{\infty} x\ln(15x)\,dx$	6) $\int_0^7 (2x+3)\left(x^2+3x\right)^{-1/2}\,dx$
7) $\int_0^{16} \frac{1}{\sqrt{16-x}}\,dx$	8) $\int_{-\infty}^{1} \theta e^{\theta}\,d\theta$

Answers

1) $\frac{5}{2}$	2) Diverges	3) -2	4) $-4\ln\frac{7}{8}$
5) Diverges	6) $\sqrt{70}$	7) 8	8) 0

AP Calculus BC – Worksheet 47 **Integration using division / Review**

Evaluate the integral or state that it diverges.

1) $\int \frac{x+4}{x+6}dx$	2) $\int \frac{t^3-4}{t+2}dt$
3) $\int_{-1}^{1} x^{-\frac{1}{3}}dx$	4) $\int_{14}^{\infty} \frac{4}{v^2-v}dv$
5) $\int_{-\infty}^{\infty} 16x^7 e^{-x^8}dx$	6) $\int_{5}^{11} \frac{18x}{9x^2+6}dx$
7) $\int x\sin\left(\frac{1}{5}x\right)dx$	8) $\int_{1}^{e^4} x^3 \ln x dx$
9) $\int 9x^3 e^{3x}dx$	10) $\int \frac{10r^4}{\sqrt{1-r^5}}dr$
11) $\int_{0}^{\pi/6} \cos^{-8} 2x \sin 2x dx$	

Answers

1) $x-2\ln\lvert x+6\rvert+C$	2) $\frac{1}{3}t^3-t^2+4t-12\ln\lvert t+2\rvert+C$	3) 0	4) $-4\ln\frac{13}{14}$
5) 0	6) $\ln\frac{1095}{231}=\ln\frac{365}{77}$	7) $-5x\cos\left(\frac{1}{5}x\right)+25\sin\left(\frac{1}{5}x\right)+C$	8) $\frac{15}{16}e^6+\frac{1}{16}$
9) $3x^3e^{3x}-3x^{2e^{3x}}+2xe^{3x}-\frac{54}{81}e^{3x}+C$		10) $-4\left(1-r^5\right)^{1/2}+C$	11) $\frac{2^7-1}{14}$

AP Calculus BC – Worksheet 48 **Integration Techniques Review**

1	The velocity of an object is given by $v(t)=7t^6-4t^2+12$ with $s(1)=24$. Find $s(t)$.
2	Evaluate $\int \frac{4x^4+3}{4x^5+15x+2}dx$
3	Use substitution to evaluate $\int_0^1 \left(x^3+x\right)\left(x^4+2x^2+9\right)^{1/2}dx$
4	$\int x^2 \sin x dx =$
5	$\int_1^2 \left(9x^2-4x+1\right)\ln x dx =$
6	$\int \frac{4}{x^2+3x+2}dx =$
7	$\int_0^1 \frac{1}{1-x}dx =$
8	$\int_1^\infty \frac{x}{\left(1+x^2\right)^2}dx =$
9	$\int_{-1}^1 \frac{3x^2+2x+1}{x+4}dx =$
10	If $\int_{-2}^8 \left(3g(x)+2\right)dx=35$ and $\int_5^{-2} g(x)dx=-12$, then $\int_5^8 g(x)dx =$
11	For values of h very close to zero, find a function that best approximates $f(x)=\frac{\cos(x+h)-\cos x}{h}$.
12	$\lim_{x\to\infty} \frac{\ln(3x+5)}{\ln\left(2x^2-1\right)} =$